Lecture 4: Sensor interface circuits

- **Review of circuit theory**
  - Voltage, current and resistance
  - Capacitance and inductance
  - Complex number representations
- **Measurement of resistance**
  - Voltage dividers
  - Wheatstone Bridge
  - Temperature compensation for strain gauges
- **AC bridges**
  - Measurement of capacitance
  - Measurement of impedance

Voltage, current, resistance and power

- **Voltage**
  - The voltage between two points is the energy required to move a unit of positive charge from a lower to a higher potential. Voltage is measured in Volts (V)
- **Current**
  - Current is the rate of electric charge through a point. The unit of measure is the Ampere or Amp (A)
- **Resistance**
  - Given a piece of conducting material connected to a voltage difference V, which drives through it a current *I*, the resistance is defined as

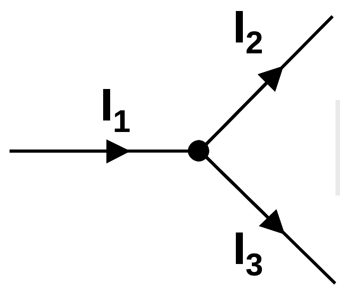
$$R = \frac{V}{I}$$

    - As you will recall, this is known as **Ohm's Law**
    - An element whose resistance is constant for all values of V is called an *ohmic* resistor
  - Series and parallel resistors…
- **Power**
  - The power dissipated by a resistor is

$$P = VI = \frac{V^2}{R} = I^2R$$

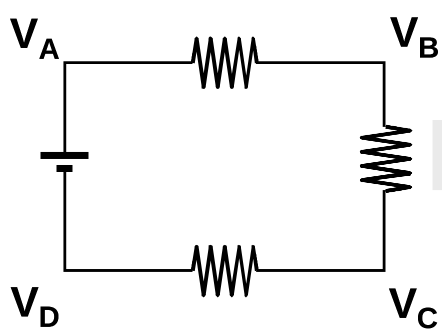
Kirchhoff's Laws

- **1^{st} Law (for nodes)**
  - The algebraic sum of the currents into any node of a circuit is zero
    - Or, the sum of the currents entering equals the sum of the currents leaving
    - Thus, elements in series have the same current flowing through them



$$I_1=I_2+I_3$$

- **2^{nd} Law (for loops)**
  - The algebraic sum of voltages in a loop is zero
    - Thus, elements in parallel have the same voltage across them.



$$V_{AB}+V_{BC}+V_{CD}+V_{DA}=0$$

Capacitors and inductors

- **A capacitor is an element capable of storing charge**
  - The amount of charge is proportional to the voltage across the capacitor

$$Q = CV$$

    - C is known as the capacitance (measured in *Farads*)
  - Taking derivatives

$$\frac{dQ}{dt} = \frac{d(CV)}{dt} \Rightarrow I = C\frac{dV}{dt}$$

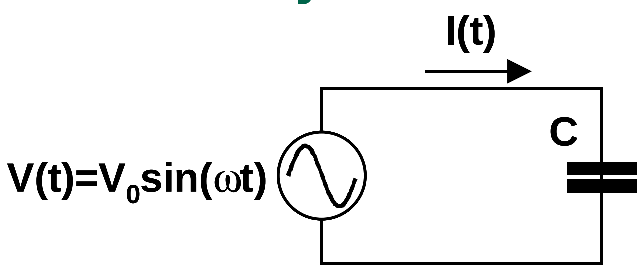
  - Therefore, a capacitor is an element whose rate of voltage change is proportional to the current through it
- **Similarly, an inductor is an element whose rate of current change is proportional to the voltage applied across it**

$$V = L\frac{dI}{dt}$$

    - L is called the inductance and is measured in Henrys

Frequency analysis

- **Consider a capacitor driven by a sine wave voltage**



- The current through the capacitor is

$$I = C\frac{dV}{dt} = C\frac{d}{dt}\left(V_0\sin(\omega t)\right) = C\omega V_0\cos(\omega t)$$

  - Therefore, the current phase-leads the voltage by 90^0 and the ratio of amplitudes is

$$\frac{|V(t)|}{|I(t)|} = \frac{1}{C\omega}$$

  - What happens when the voltage is a DC source?

Voltages as complex numbers

- **At this point it is convenient to switch to a complex-number representation of signals**
  - Recall that $\boldsymbol{e^{j\varphi}=\cos\varphi+j\sin\varphi}$

$$\mathbf{V_0\cos(\omega t+\varphi)}$$
$$\Downarrow$$
$$\mathbf{V_0 e^{j\varphi}}$$

Circuit voltage versus time: $\mathbf{V(t)=V_0\cos(\omega t+\varphi)}$

Complex number representation: $\mathbf{V_0 e^{j\varphi}=a+jb}$

Multiply by $\mathbf{e^{j\varphi}}$ and take real part

From [HH89]

- **Applying this to the capacitor V(t)/I(t) relationship**

$$I = C\frac{dV}{dt} = C\omega V_0\cos(\omega t)$$
$$\Downarrow$$
$$\frac{V}{I} = \frac{V_0\sin(\omega t)}{C\omega V_0\cos(\omega t)} = \frac{\cos(\omega t-\pi/2)}{C\omega e^{j0}} = \frac{e^{-j\pi/2}}{C\omega} = \frac{1}{C\omega e^{-j\pi/2}} = \frac{1}{j\omega C}$$

Impedance

- **Impedance (Z) is a generalization of resistance for circuits that have capacitors and inductors**
  - Capacitors and inductors have **reactance**, while resistors have **resistance**

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$Z_L = j\omega L$$

$$Z_R = R$$

- **Ohm's Law generalized**

$$\frac{V}{I} = Z$$

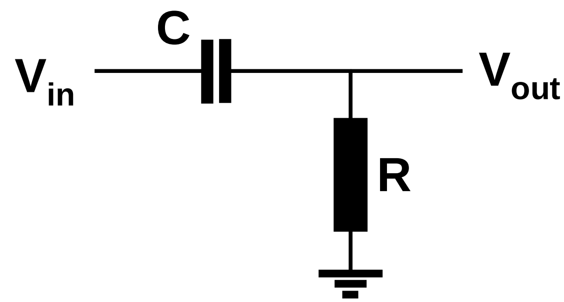
- **Impedance in series and parallel**

$$Z_S = Z_1 + Z_2 + \cdots Z_N$$

$$\frac{1}{Z_P} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}$$

Example: High-pass filter

- **High pass filter**
  - The current through cap and resistor is



$$I = \frac{V_{in}}{Z} = \frac{V_{in}}{R + \frac{1}{j\omega C}}$$

  - The output voltage is equal to the voltage differential across the resistor

$$V_{out} = RI = R\frac{V_{in}}{R + \frac{1}{j\omega C}}$$

  - If we focus on amplitude and ignore phase

$$|V_{out}| = R\frac{|V_{in}|}{\left|R + \frac{1}{j\omega C}\right|} = R\frac{|V_{in}|}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = |V_{in}|\frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}$$

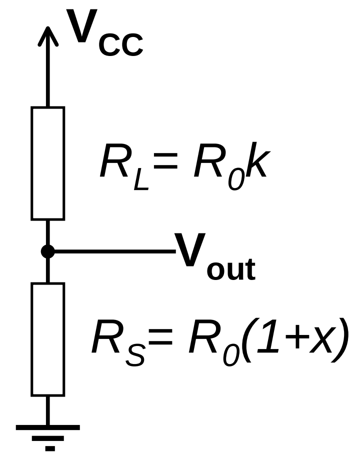
    - Asymptotic behavior…
    - Corner frequency $\omega_{CORNER} = \frac{1}{RC} \Rightarrow 20\log_{10}\frac{|V_{out}|}{|V_{in}|} = 20\log_{10}\frac{1}{\sqrt{1+1}} = \text{-}3.010\,\text{dB}$

Measurement circuits

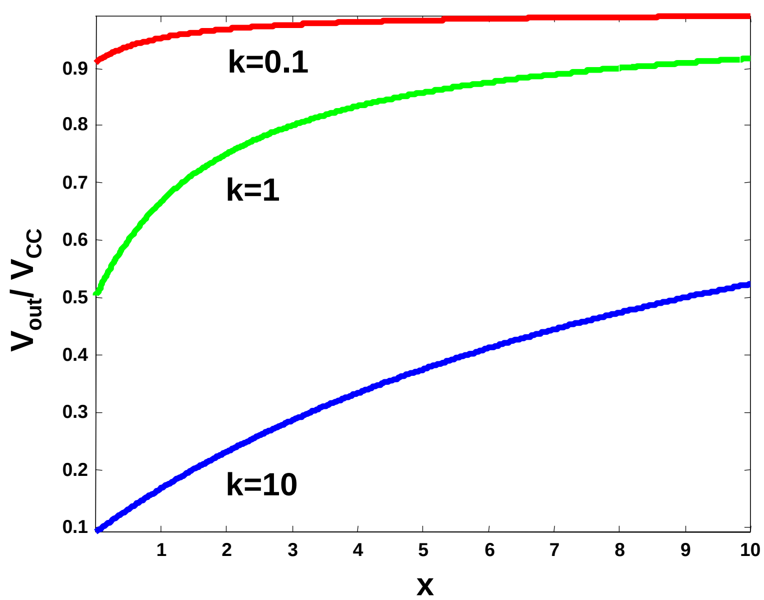
- **Resistance measurements**
  - Voltage divider (half-bridge)
  - Wheatstone bridge
- **A.C. bridges**
  - Measurement of capacitance
  - Measurement of impedance

Voltage divider

- **Assumptions**
  - Interested in measuring the fractional change in resistance ***x*** of the sensor: $R_S=R_0(1+x)$
    - R_0 is the sensor resistance in the absence of a stimuli
  - Load resistor expressed as $R_L=R_0k$ for convenience
- **The output voltage of the circuit is**



$$V_{out} = V_{CC}\frac{R_S}{R_S+R_L} =$$
$$= V_{CC}\frac{R_0(1+x)}{R_0(1+x)+R_0k} = V_{CC}\frac{1+x}{1+x+k}$$



- **Questions**
  - What if we reverse R_S and R_L?
  - How can we recover R_S from V_{out}?

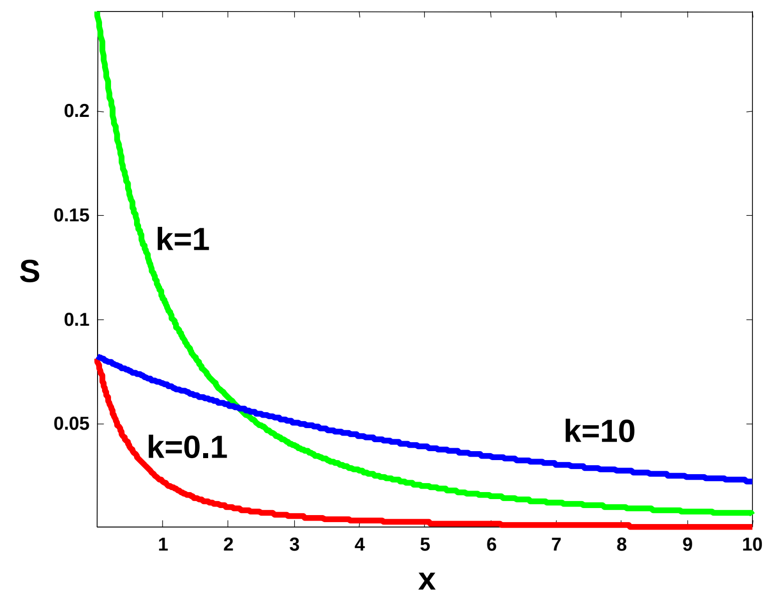
Voltage divider

- **What is the sensitivity of this circuit?**

$$S = \frac{dV_{out}}{dx} = \frac{d}{dx}\left(V_{CC}\frac{1+x}{1+x+k}\right) =$$

$$= V_{CC}\frac{(1+x+k)-(1+x)}{(1+x+k)^2} =$$

$$= V_{CC}\frac{k}{(1+x+k)^2}$$



- **For which R_L do we achieve maximum sensitivity?**

$$\frac{dS}{dk} = 0 \Rightarrow \frac{d}{dk}\left(V_{CC}\frac{k}{(1+x+k)^2}\right) = 0 \Rightarrow \frac{(1+x+k)^2 - k2(1+x+k)}{(1+x+k)^2} = 0 \Rightarrow \mathbf{k} = \mathbf{1} + \mathbf{x}$$

  - This is, the sensitivity is maximum when R_L=R_S

Wheatstone bridge

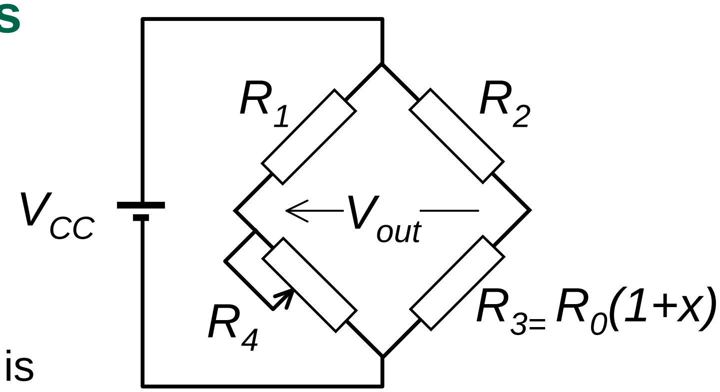
- **A circuit that consists of two dividers**
  - A reference voltage divider (left)
  - A sensor voltage divider
- **Wheatstone bridge operating modes**
  - Null mode
    - R_4 adjusted until the balance condition is met:

$$V_{out} = 0 \Leftrightarrow R_3 = R_4 \frac{R_2}{R_1}$$

      - Advantage: measurement is independent of fluctuations in V_{CC}
  - Deflection mode
    - The unbalanced voltage V_{out} is used as the output of the circuit

$$V_{out} = V_{CC}\left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_3 + R_4}\right)$$

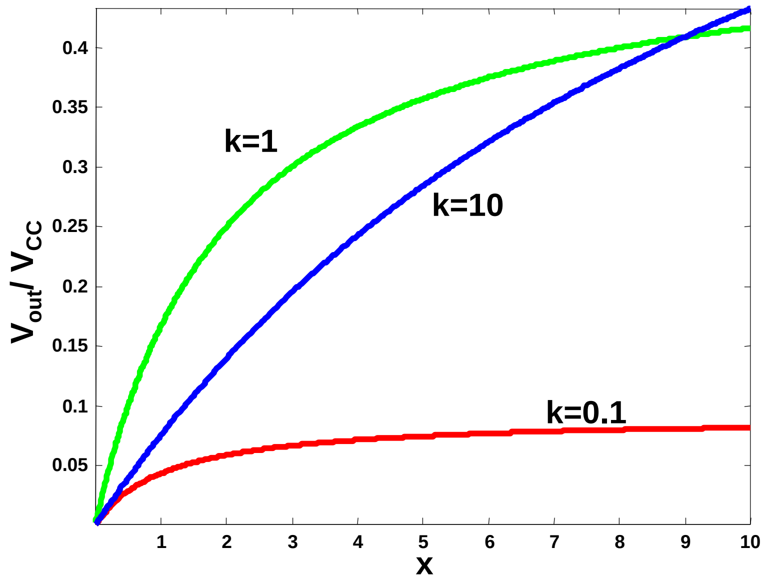
      - Advantage: speed



Wheatstone bridge

- **Assumptions**
  - Want to measure sensor fractional resistance changes $R_S=R_0(1+x)$
  - Bridge is operating near the balance condition:

$$k = \frac{R_1}{R_4} = \frac{R_2}{R_0}$$



- **The output voltage becomes**

$$V_{out} = V_{CC}\left(\frac{R_0(1+x)}{R_0k+R_0(1+x)} - \frac{R_4}{R_4k+R_4}\right) =$$

$$= V_{CC}\left(\frac{(1+x)}{k+(1+x)} - \frac{1}{k+1}\right) = V_{CC}\frac{kx}{(1+k)(1+k+x)}$$

Wheatstone bridge

- **What is the sensitivity of the Wheatstone bridge?**

$$S = \frac{dV_{out}}{dx} = V_{CC}\frac{d}{dx}\left(\frac{kx}{(1+k)(1+k+x)}\right) =$$

$$= V_{CC}\frac{k(1+k)(1+k+x) - kx(1+k)}{(1+k)^2(1+k+x)^2} =$$

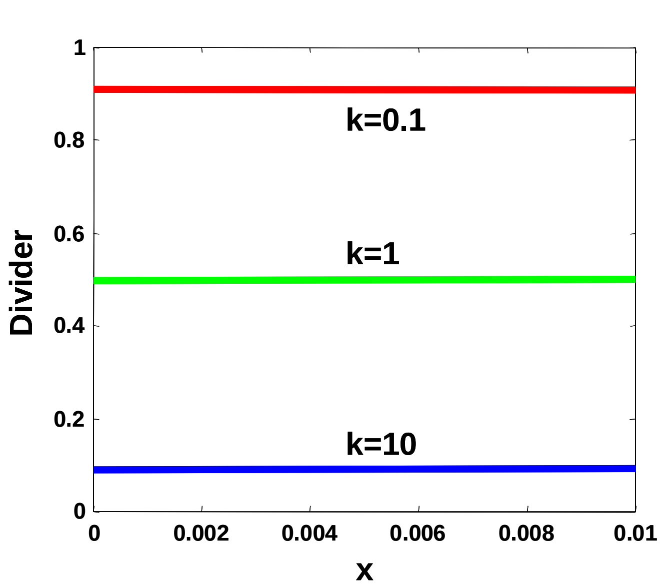
$$= V_{CC}\frac{k}{(1+k+x)^2}$$

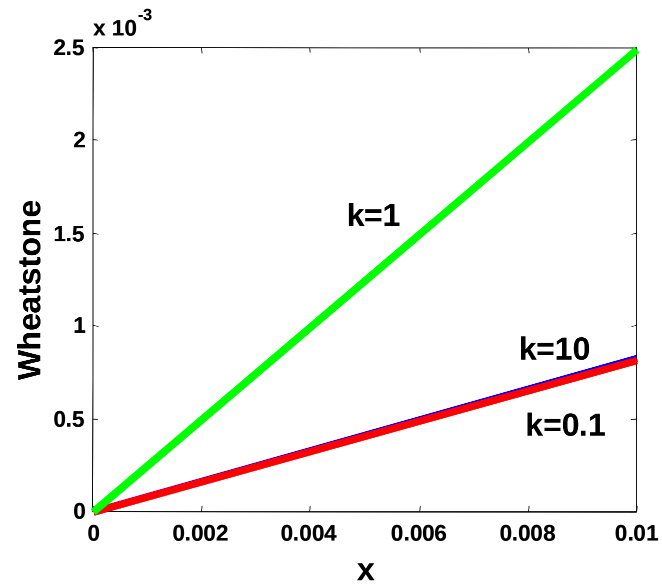
  - The sensitivity of the Wheatstone bridge is the same as that of a voltage divider
    - You can think of the Wheatstone bridge as a DC offset removal circuit

- **So what are the advantages, if any, of the Wheatstone bridge?**

Voltage divider vs. Wheatstone for small x

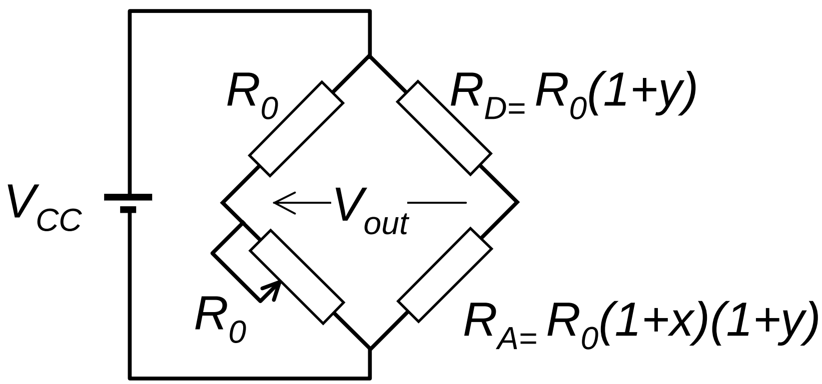
- **The figures below show the output of both circuits for small fractional resistance changes**
  - The voltage divider has a large DC offset compared to the voltage swing, which makes the curves look “flat” (zero sensitivity)
    - Imagine measuring the height of a person standing on top of a tall building by running a large tape measure from the street
  - The sensitivity of both circuits is the same!
    - However, the Wheatstone bridge sensitivity can be boosted with a gain stage
      - Assuming that our DAQ hardware dynamic range is 0-5VDC, $0<x<0.01$ and k=1, estimate the maximum gain that could be applied to each circuit

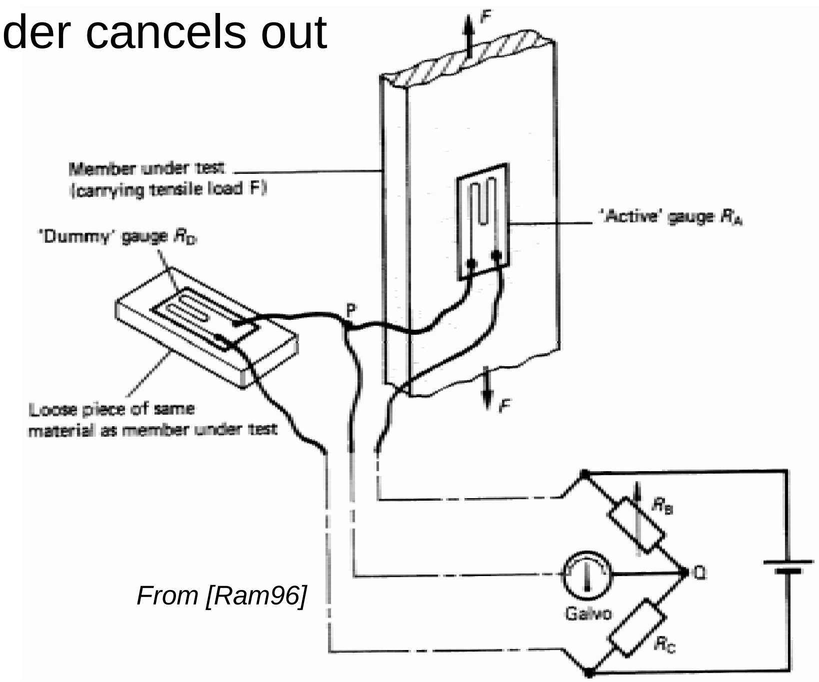




Compensation in a Wheatstone bridge

- **Strain gauges are quite sensitive to temperature**
  - A Wheatstone bridge and a dummy strain gauge may be used to compensate for this effect
    - The "active" gauge R_A is subject to temperature (x) and strain (y) stimuli
    - The dummy gauge R_D, placed near the "active" gauge, is only subject to temperature
  - The gauges are arranged according to the figures below
  - The effect of (1+y) on the right divider cancels out

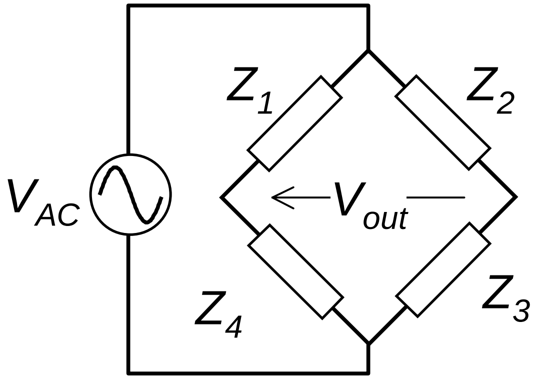




From [Ram96]

AC bridges

- **The structure of the Wheatstone bridge can be used to measure capacitive and inductive sensors**
  - Resistance replaced by generalized impedance
  - DC bridge excitation replaced by an AC source
- **The balance condition becomes**



$$\frac{Z_1}{Z_4} = \frac{Z_2}{Z_3}$$

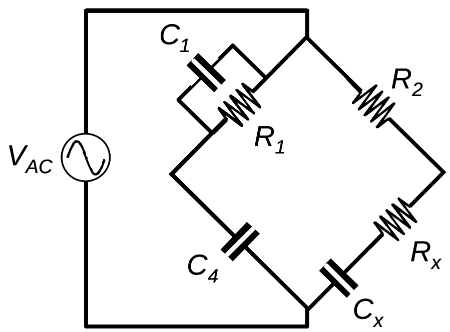
  - which yields two equalities, for real and imaginary components

$$R_1R_3 - X_1X_3 = R_2R_4 - X_2X_4$$
$$R_1X_3 + X_1R_3 = R_2X_4 + X_2R_4$$

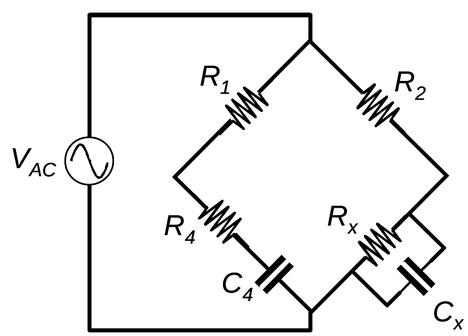
- **There is a large number of AC bridge arrangements**
  - These are named after their respective developer

AC bridges

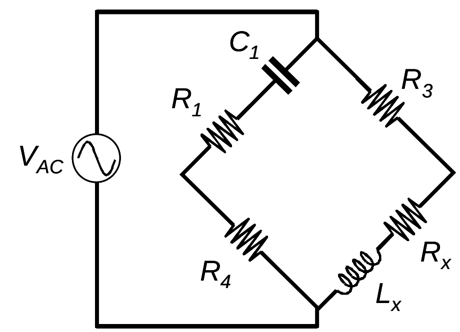
- **Capacitance measurement**
  - Schering bridge
  - Wien bridge
- **Inductance measurement**
  - Hay bridge
  - Owen bridge



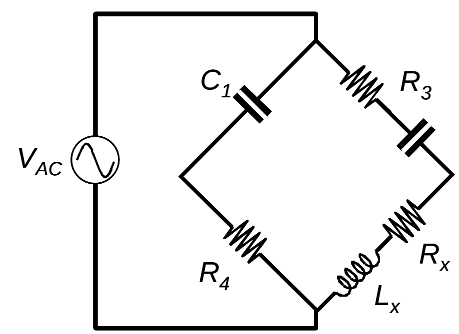
Schering



Wien



Hay



Owen

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