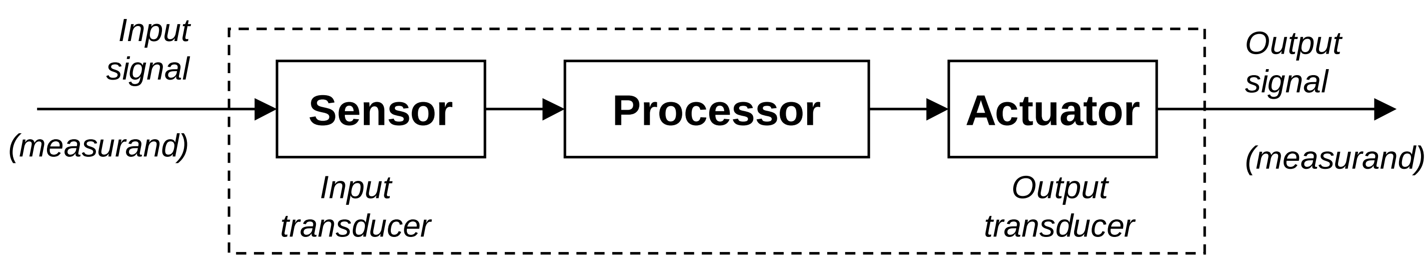
Lecture 2: Sensor characteristics

- Transducers, sensors and measurements
- Calibration, interfering and modifying inputs
- Static sensor characteristics
- Dynamic sensor characteristics

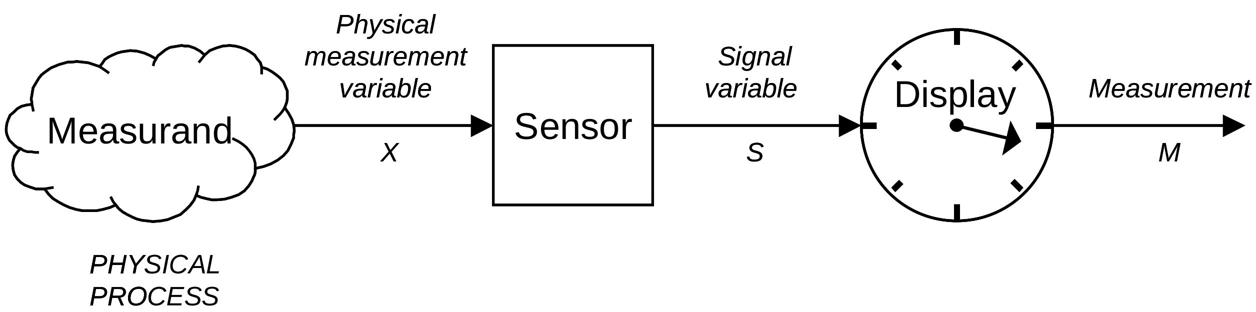
Transducers: sensors and actuators

- **Transducer**
  - A device that converts a signal from one physical form to a corresponding signal having a different physical form
    - Physical form: mechanical, thermal, magnetic, electric, optical, chemical…
  - Transducers are ENERGY CONVERTERS or MODIFIERS
- **Sensor**
  - A device that receives and responds to a signal or stimulus
    - This is a broader concept that includes the extension of our perception capabilities to acquire information about physical quantities
- **Transducers: sensors and actuators**
  - Sensor: an input transducer (i.e., a microphone)
  - Actuator: an output transducer (i.e., a loudspeaker)



Measurements

- **A simple instrument model**

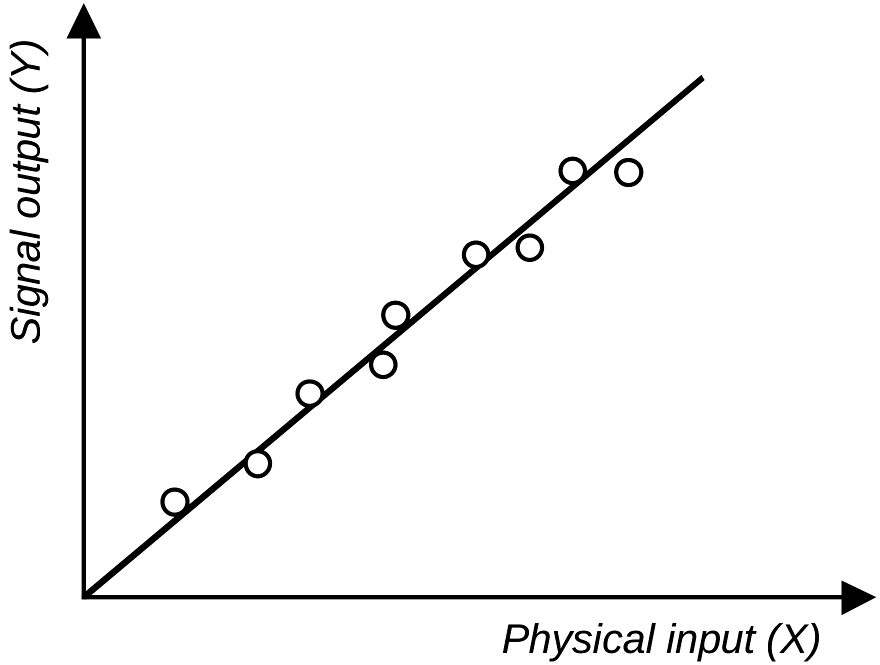


  - A observable variable X is obtained from the measurand
    - X is related to the measurand in some KNOWN way (i.e., measuring mass)
  - The sensor generates a signal variable that can be manipulated:
    - Processed, transmitted or displayed
  - In the example above the signal is passed to a display, where a measurement can be taken

- **Measurement**
  - The process of comparing an unknown quantity with a standard of the same quantity (measuring length) or standards of two or more related quantities (measuring velocity)

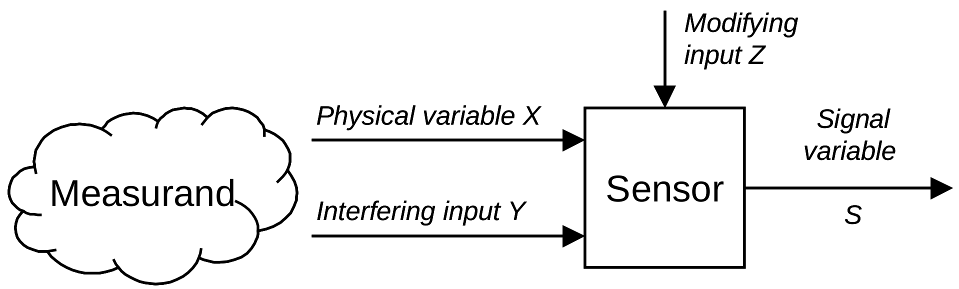
Calibration

- **The relationship between the physical measurement variable (X) and the signal variable (S)**
  - A sensor or instrument is calibrated by applying a number of KNOWN physical inputs and recording the response of the system

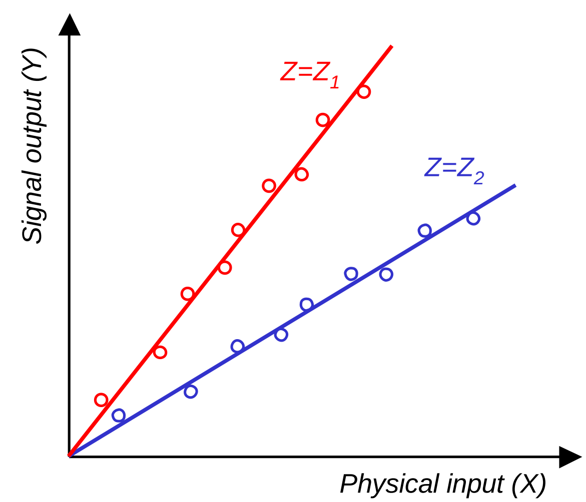


Additional inputs

- **Interfering inputs (Y)**
  - Those that the sensor to respond as the linear superposition with the measurand variable X
    - Linear superposition assumption: S(aX+bY)=aS(X)+bS(Y)



- **Modifying inputs (Z)**
  - Those that change the behavior of the sensor and, hence, the calibration curve
    - Temperature is a typical modifying input



Sensor characteristics [PAW91, Web99]

- **Static characteristics**
  - The properties of the system after all transient effects have settled to their final or steady state
    - **Accuracy**
    - **Discrimination**
    - **Precision**
    - Errors
    - Drift
    - Sensitivity
    - Linearity
    - Hystheresis (backslash)
- **Dynamic characteristics**
  - The properties of the system transient response to an input
    - Zero order systems
    - First order systems
    - Second order systems

Accuracy, discrimination and precision

- **Accuracy is the capacity of a measuring instrument to give RESULTS close to the TRUE VALUE of the measured quantity**
  - Accuracy is related to the <u>bias</u> of a set of measurements
  - (IN)Accuracy is measured by the absolute and relative errors

$$\textbf{ABSOLUTE ERROR} = \textbf{RESULT - TRUE VALUE}$$

$$\textbf{RELATIVE ERROR} = \frac{\textbf{ABSOLUTE ERROR}}{\textbf{TRUE VALUE}}$$

- More on errors in a later slide

- **Discrimination is the minimal change of the input necessary to produce a detectable change at the output**
  - Discrimination is also known as RESOLUTION
  - When the increment is from zero, it is called THRESHOLD

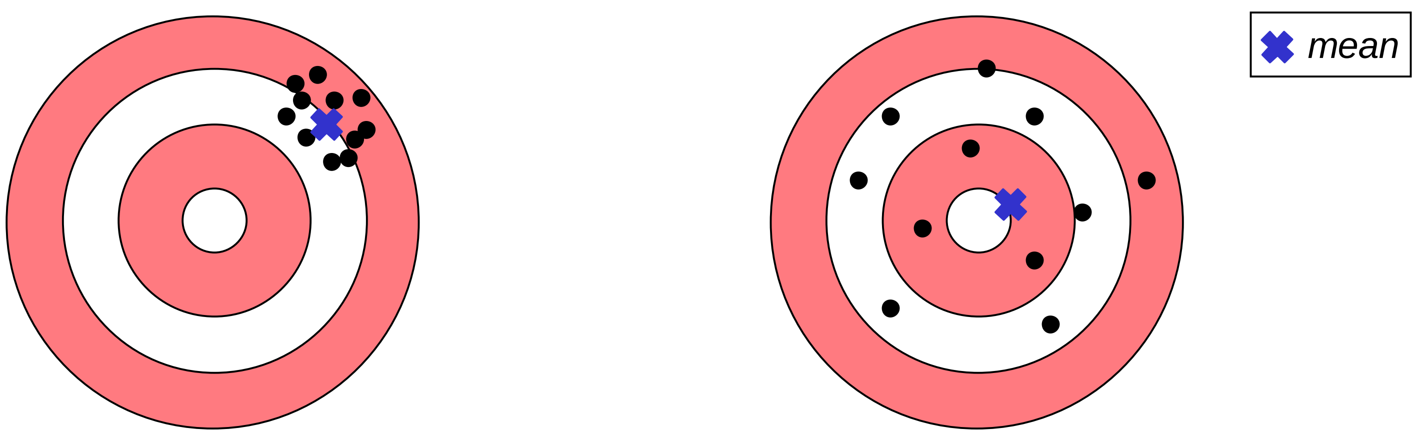
Precision

- **The capacity of a measuring instrument to give the same reading when repetitively measuring the same quantity under the same prescribed conditions**
  - Precision implies agreement between successive readings, NOT closeness to the true value
    - Precision is related to the variance of a set of measurements
  - Precision is a necessary but not sufficient condition for accuracy
- **Two terms closely related to precision**
  - Repeatability
    - The precision of a set of measurements taken over a short time interval
  - Reproducibility
    - The precision of a set of measurements BUT
      - taken over a long time interval or
      - Performed by different operators or
      - with different instruments or
      - in different laboratories

Example

■ Shooting darts

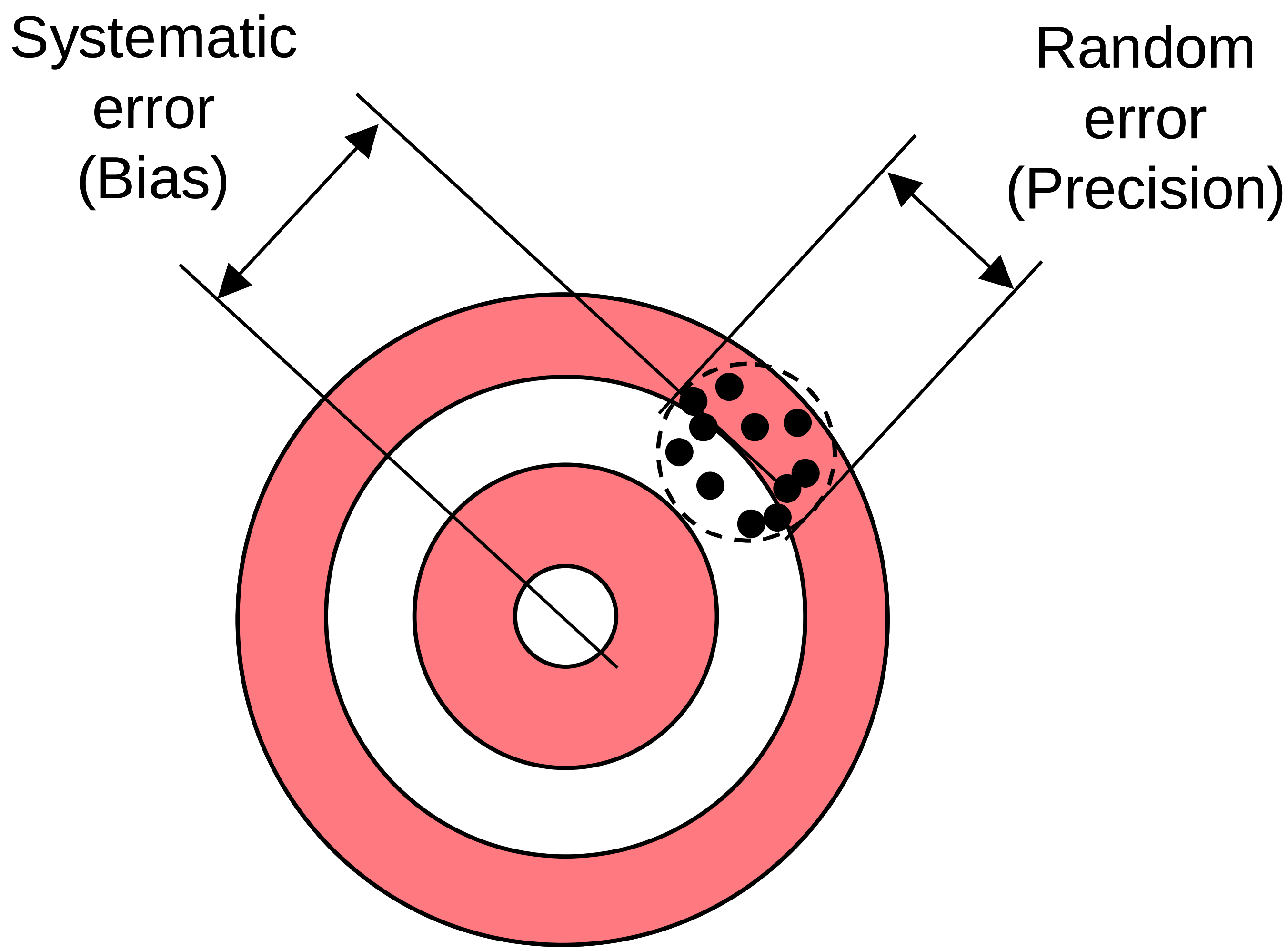
- Discrimination
  - The size of the hole produced by a dart
- Which shooter is more accurate?
- Which shooter is more precise?



Accuracy and errors

- **Systematic errors**
  - Result from a variety of factors
    - Interfering or modifying variables (i.e., temperature)
    - Drift (i.e., changes in chemical structure or mechanical stresses)
    - The measurement process changes the measurand (i.e., loading errors)
    - The transmission process changes the signal (i.e., attenuation)
    - Human observers (i.e., parallax errors)
  - Systematic errors can be corrected with COMPENSATION methods (i.e., feedback, filtering)
- **Random errors**
  - Also called NOISE: a signal that carries no information
  - True random errors (white noise) follow a Gaussian distribution
  - Sources of randomness:
    - Repeatability of the measurand itself (i.e., height of a rough surface)
    - Environmental noise (i.e., background noise picked by a microphone)
    - Transmission noise (i.e., 60Hz hum)
  - Signal to noise ratio (SNR) should be >>1
    - With knowledge of the signal characteristics it may be possible to interpret a signal with a low SNR (i.e., understanding speech in a loud environment)

Example: systematic and random errors

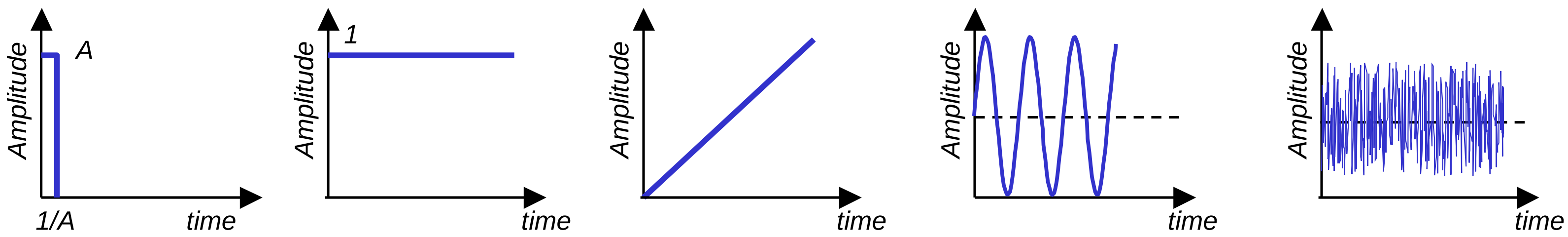


More static characteristics

- **Input range**
  - The maximum and minimum value of the physical variable that can be measured (i.e., -40F/100F in a thermometer)
  - Output range can be defined similarly
- **Sensitivity**
  - The slope of the calibration curve y=f(x)
    - An ideal sensor will have a large and constant sensitivity
  - Sensitivity-related errors: saturation and “dead-bands”
- **Linearity**
  - The closeness of the calibration curve to a specified straight line (i.e., theoretical behavior, least-squares fit)
- **Monotonicity**
  - A monotonic curve is one in which the dependent variable always increases or decreases as the independent variable increases
- **Hystheresis**
  - The difference between two output values that correspond to the same input depending on the trajectory followed by the sensor (i.e., magnetization in ferromagnetic materials)
    - Backslash: hystheresis caused by looseness in a mechanical joint

Dynamic characteristics

- **The sensor response to a variable input is different from that exhibited when the input signals are constant (the latter is described by the static characteristics)**
- **The reason for dynamic characteristics is the presence of energy-storing elements**
  - Inertial: masses, inductances
  - Capacitances: electrical, thermal
- **Dynamic characteristics are determined by analyzing the response of the sensor to a family of variable input waveforms:**
  - Impulse, step, ramp, sinusoidal, white noise…



Dynamic models

- **The dynamic response of the sensor is (typically) assumed to be linear**
  - Therefore, it can be modeled by a constant-coefficient linear differential equation

$$a_k \frac{d^k y(t)}{dt^k} + \cdots a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t)$$

  - In practice, these models are confined to zero, first and second order. Higher order models are rarely applied
- **These dynamic models are typically analyzed with the Laplace transform, which converts the differential equation into a polynomial expression**
  - Think of the Laplace domain as an extension of the Fourier transform
    - Fourier analysis is restricted to sinusoidal signals
      - $x(t) = \sin(\omega t) = e^{-j\omega t}$
    - Laplace analysis can also handle exponential behavior
      - $x(t) = e^{-\sigma t}\sin(\omega t) = e^{-(\sigma + j\omega)t}$

The Laplace Transform (review)

- **The Laplace transform of a time signal y(t) is denoted by**
  - L[y(t)] = Y(s)
    - The s variable is a complex number $s=\sigma+j\omega$
      - The real component σ defines the real exponential behavior
      - The imaginary component defines the frequency of oscillatory behavior
- **The fundamental relationship is the one that concerns the transformation of differentiation**

$$L\left[\frac{d}{dt}y(t)\right]=sY(s)-f(0)$$

- **Other useful relationships are**

$$\text{Impulse:}\quad L[\delta(t)]=1$$

$$\text{Step:}\quad L[u(t)]=\frac{1}{s}$$

$$\text{Ramp:}\quad L[r(t)]=\frac{1}{s^2}$$

$$\text{Decay:}\quad L[\exp(at)]=(s-a)^{-1}$$

$$\text{Sine:}\quad L[\sin(\omega t)]=\frac{\omega}{s^2+\omega^2}$$

$$\text{Cosine:}\quad L[\cos(\omega t)]=\frac{s}{s^2+\omega^2}$$

The Laplace Transform (review)

- **Applying the Laplace transform to the sensor model yields**

$$L\left[a_k \frac{d^k y}{dt^k} + \cdots a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_o y(t) = x(t)\right]$$

$$\Downarrow$$

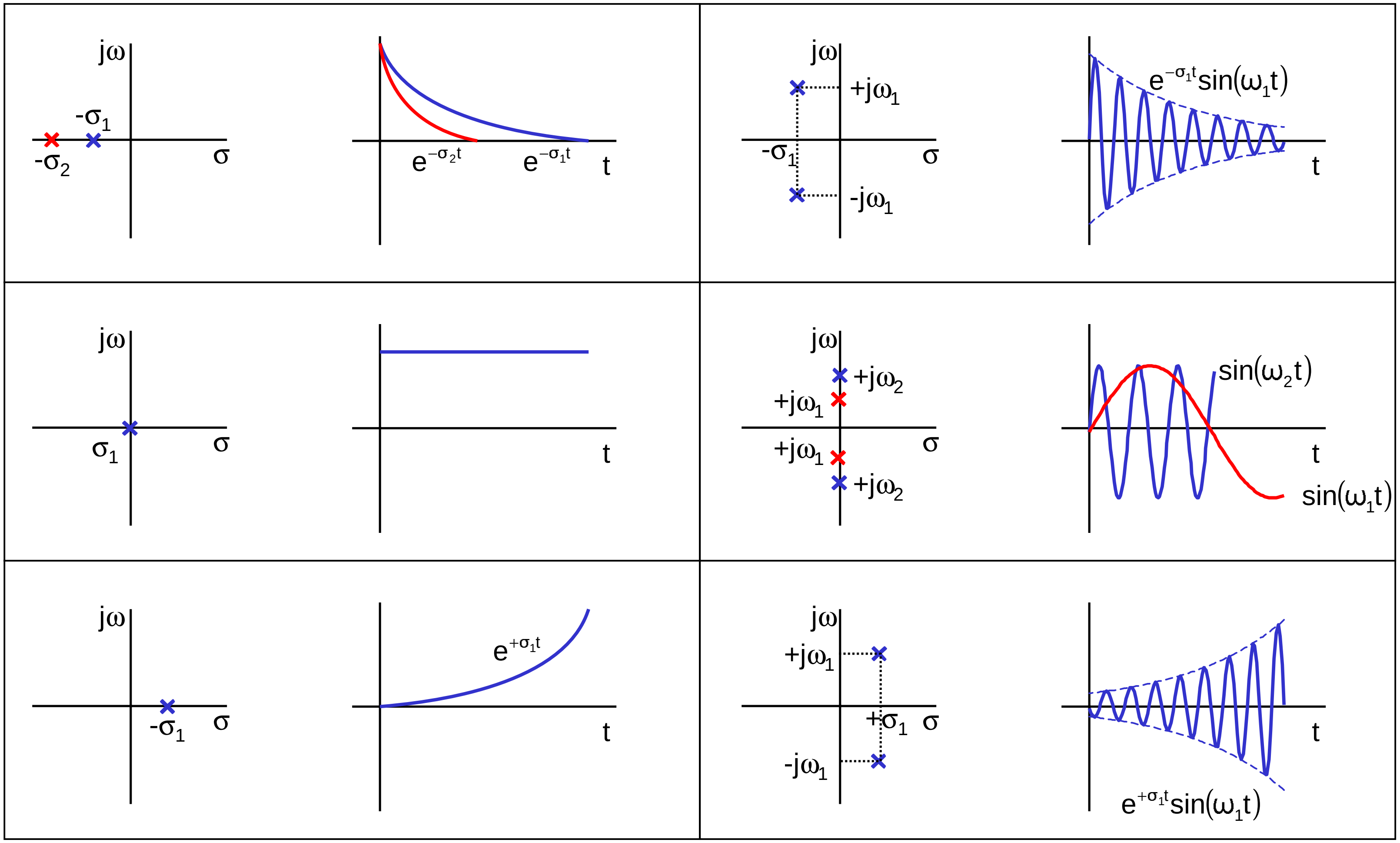
$$\left(a_k s^k + \cdots a_2 s^2 + a_1 s + a_o\right) Y(s) = X(s)$$

$$\Downarrow$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{a_k s^k + \cdots a_2 s^2 + a_1 s + a_o}$$

  - G(s) is called the transfer function of the sensor
- **The position of the poles of G(s) -zeros of the denominator- in the s-plane determines the dynamic behavior of the sensor such as**
  - Oscillating components
  - Exponential decays
  - Instability

Pole location and dynamic behavior

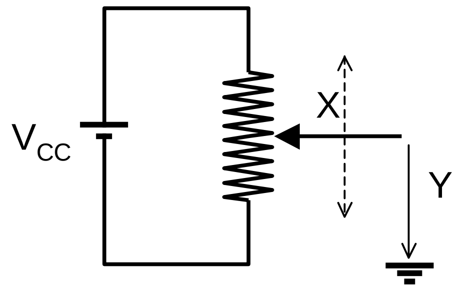


Zero-order sensors

- **Input and output are related by an equation of the type**

$$y(t) = k \cdot x(t) \Rightarrow \frac{Y(s)}{X(s)} = k$$

- Zero-order is the desirable response of a sensor
  - No delays
  - Infinite bandwidth
  - The sensor only changes the amplitude of the input signal
- Zero-order systems do not include energy-storing elements
- Example of a zero-order sensor
  - A potentiometer used to measure linear and rotary displacements
    - This model would not work for fast-varying displacements



First-order sensors

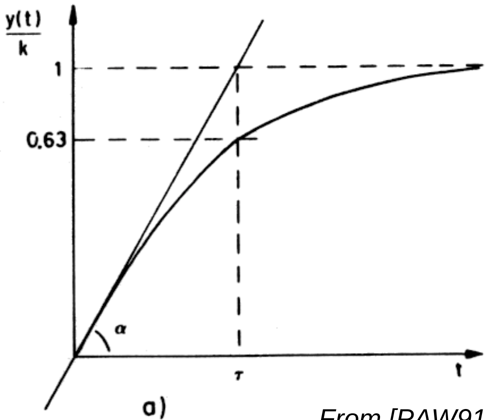
- **Inputs and outputs related by a first-order differential equation**

$$a_1 \frac{dy}{dt} + a_0 y(t) = x(t) \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{a_1 s + a_0} = \frac{k}{\tau s + 1}$$

- First-order sensors have one element that stores energy and one that dissipates it
- Step response
  - $y(t) = Ak(1-e^{-t/\tau})$
    - A is the amplitude of the step
    - k ($=1/a_0$) is the static gain, which determines the static response
    - τ ($=a_1/a_0$) is the time constant, which determines the dynamic response
- Ramp response
  - $y(t) = Akt - Ak\tau u(t) + Ak\tau e^{-t/\tau}$
- Frequency response
  - Better described by the amplitude and phase shift plots

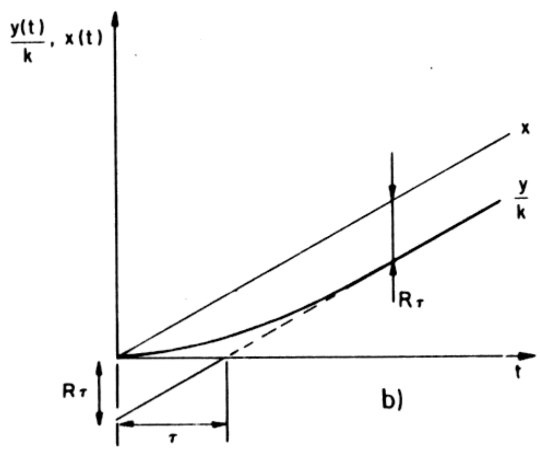
First-order sensor response

- **Step response**

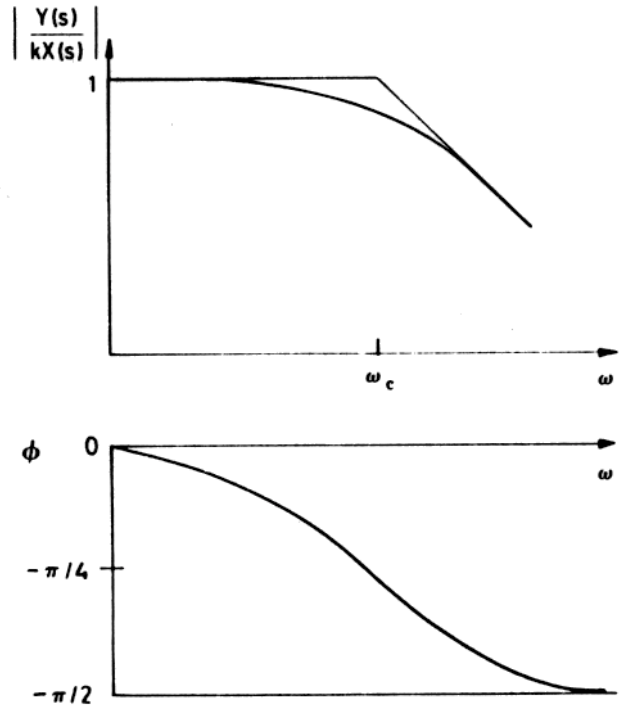


From [PAW91]

- **Ramp response**

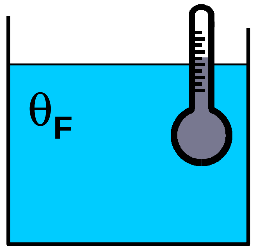


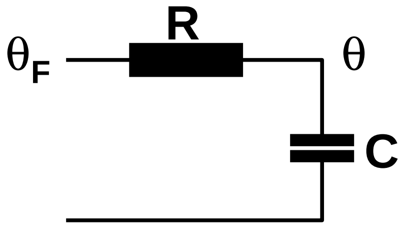
- **Frequency response**
  - Corner frequency $\omega_c = 1/\tau$
  - Bandwidth



Example of a first-order sensor

- **A mercury thermometer immersed into a fluid**
  - What type of input was applied to the sensor?
  - Parameters
    - C: thermal capacitance of the mercury
    - R: thermal resistance of the glass to heat transfer
    - θ_F: temperature of the fluid
    - $\theta(t)$: temperature of the thermometer
  - The equivalent circuit is an RC network





- **Derivation**
  - Heat flow through the glass $(\theta_F - \theta(t))/R$
  - Temperature of the thermometer rises as $\frac{d\theta(t)}{dt} = \frac{\theta_F - \theta(t)}{RC}$
  - Taking the Laplace transform

$$s\,\theta(s) = \frac{\theta_F(s) - \theta(s)}{RC} \Rightarrow (RCs+1)\,\theta(s) = \theta_F(s) \Rightarrow$$

$$\Rightarrow \theta(s) = \frac{\theta_F(s)}{(RCs+1)} \quad \Rightarrow \theta(t) = \theta_F\left(1 - e^{-t/RC}\right)$$

Second-order sensors

- **Inputs and outputs are related by a second-order differential equation**

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = x(t) \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{a_2 s^2 + a_1 s + a_0}$$

- We can express this second-order transfer function as

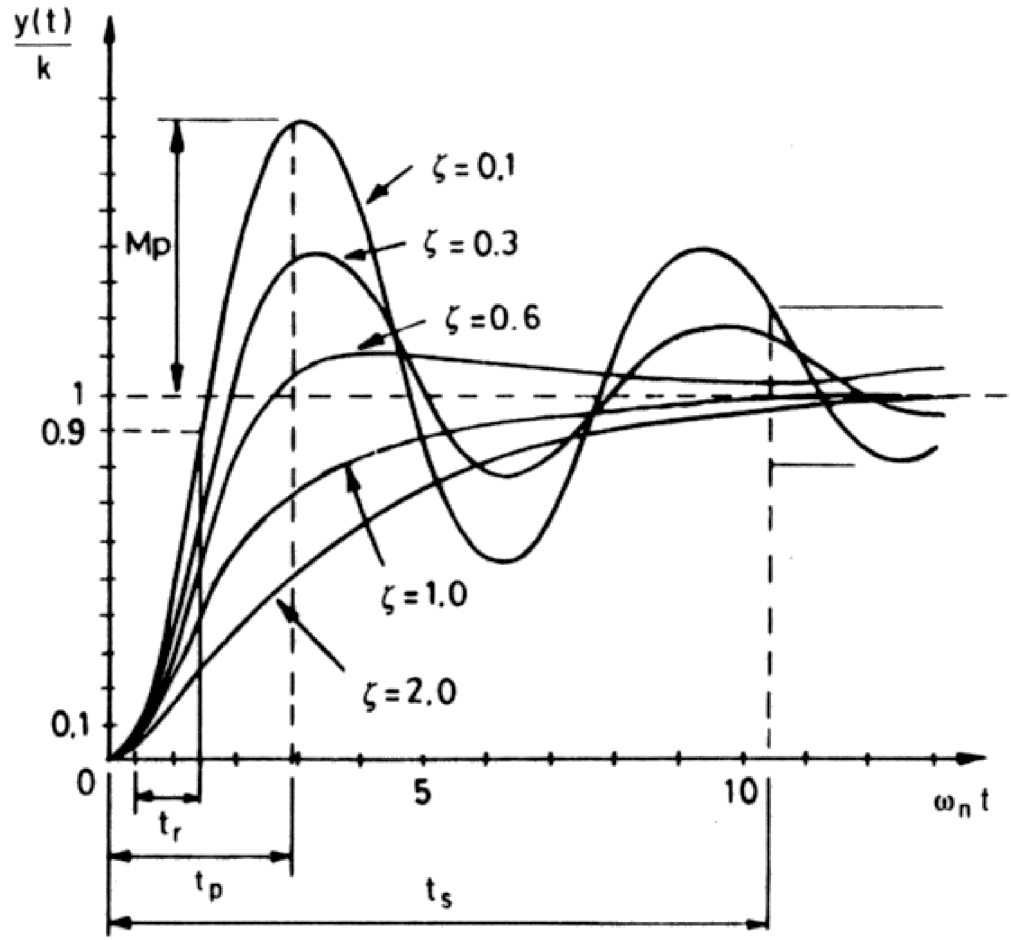
$$\frac{Y(s)}{X(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\text{with } k = \frac{1}{a_0},\ \zeta = \frac{a_1}{2\sqrt{a_0 a_1}},\ \omega_n = \sqrt{\frac{a_0}{a_2}}$$

- Where
  - k is the static gain
  - ζ is known as the damping coefficient
  - ω_n is known as the natural frequency

Second-order step response

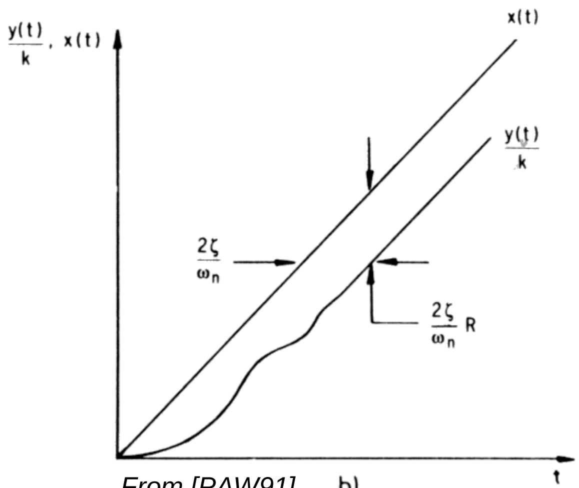
- **Response types**
  - Underdamped (ζ<1)
  - Critically damped (ζ=1)
  - Overdamped (ζ>1)
- **Response parameters**
  - Rise time (t_r)
  - Peak overshoot (M_p)
  - Time to peak (t_p)
  - Settling time (t_s)



From [PAW91]

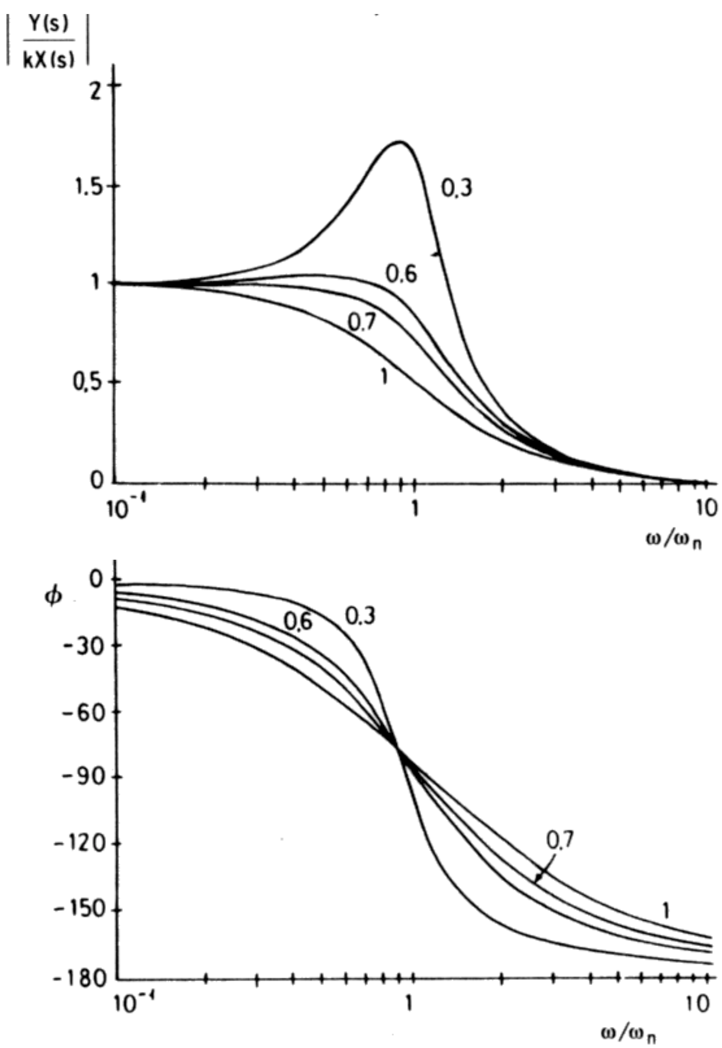
Second-order response (cont)

- **Ramp response**



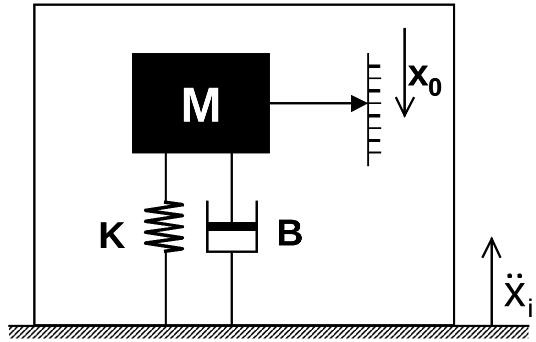
From [PAW91] b)

- **Frequency response**



Example of second-order sensors

- **A thermometer covered for protection**
  - Adding the heat capacity and thermal resistance of the protection yields a second-order system with two real poles (overdamped)
- **Spring-mass-dampen accelerometer**
  - The armature suffers an acceleration
    - We will assume that this acceleration is orthogonal to the direction of gravity
  - x_0 is the displacement of the mass M with respect to the armature
  - The equilibrium equation is:



$$M(\ddot{x}_i - \ddot{x}_0) = Kx_0 + B\dot{x}_0$$
$$\Downarrow$$
$$Ms^2X_i(s) = X_0(s)\left[K + Bs + Ms^2\right]$$
$$\Downarrow$$
$$\frac{X_0(s)}{s^2X_i(s)} = \frac{M}{K}\frac{K/M}{s^2 + s(B/M) + K/M}$$

References

[PAW91] R. Pallas-Areny and J. G. Webster, 1991, Sensors and Signal Conditioning, Wiley, New York

[Web99] J. G. Webster, 1999, The Measurement, Instrumentation and Sensors Handbook, CRC/IEEE Press , Boca Raton, FL.

[Tay97] H. R. Taylor, 1997, Data Acquisition for Sensor Systems, Chapman and Hall, London, UK.

[Fdn97] J. Fraden, 1997, Handbook of Modern Sensors. Physics, Designs and Applications, AIP, Woodbury, NY

[BW96] J. Brignell and N. White, 1996, Intelligent Sensor Systems, 2nd Ed., IOP, Bristol, UK